Calibration of hydrometers with the Cuckow method

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Introduction

The Cuckow method is a calibration method for hydrometers.

Introduction

Advantages over comparison methods with master hydrometers and many liquids:

• It uses a **single** liquid that covers all the ranges

• It doesn't need a set of fragile, breakable, master hydrometers

Experimental setup

The hydrometer is suspended from a balance.

The balance prevents the hydrometer from sinking.

The reading of the balance is a function of the density of the liquid in which the hydrometer floats freely at the same scale mark on its stem.
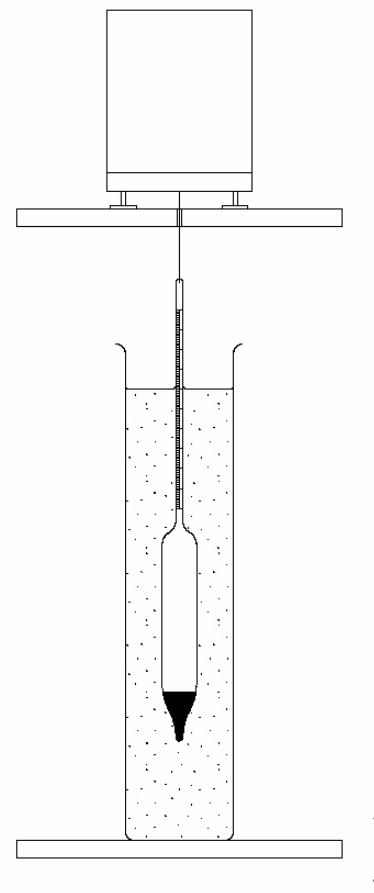
Experimental setup

The ideal is a liquid with a lower density than the scale of the hydrometer, to make sure the hydrometer sinks.

Liquids used:

• Pure water (known density, but high surface tension)

• Hydrocarbons: n-nonane, n-dodecane, … n-tetradecane, etc. (density to be calibrated, but low surface tension)
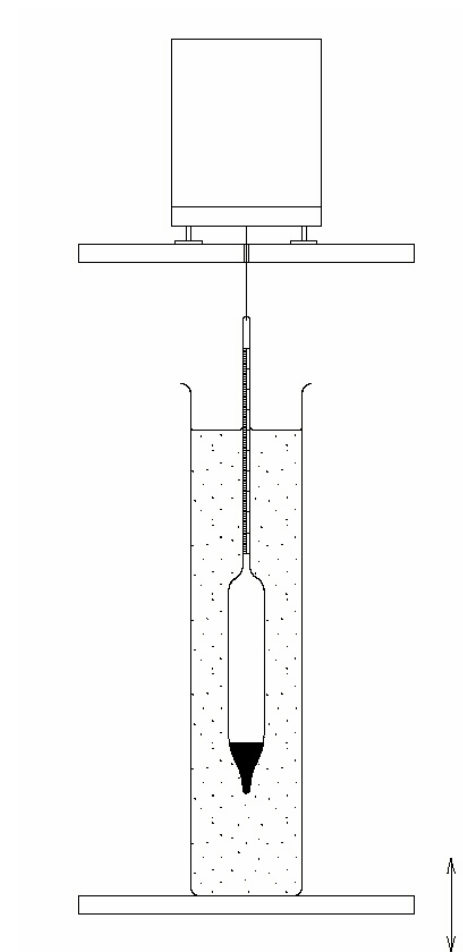
Experimental setup

If the density of the liquid is higher than the hydrometer, a ballast weight load is added to the hydrometer to make it sink.

Experimental setup

The level of immersion can be varied, so many points can be calibrated on the stem.

Steps to be performed

1. Clean the hydrometer

The cleaning of the hydrometer is done by a simple wiping with ethyl alcohol or acetone. If the surface is contaminated with hydrocarbons, as could be the case for LPG — Liquefied Petroleum Gas — hydrometers, a more complete sequence of solvents is required : varsol, naphta, and acetone. Prior to cleaning, the compatibility of the hydrometer material and the solvents must be verified.

2. Measure the stem diameters

The stem diameters are measured at the points of calibration to evaluate the stem circumferences. The diameters are measured twice at right angles with a caliper, and the mean is calculated.

The force applied downward to the hydrometer by the surface tension is proportional to the circumference.

3. Measure the hydrometer mass in air

Before doing any weighing, the balance must be exercised. It is a good habit also to calibrate it, noting the density of air at calibration.

Then, the hydrometer is weighed on the pan of the balance, producing a mass reading.

4. Supplementary ballast weight load

If the density of the hydrometer is less than that of the standard liquid, it must be loaded with a supplementary ballast weight, so that it will sink.

The value of the effective mass in liquid of the weight must be deducted from the balance readings.

Steps to be performed

5. Calibration – installation of the hydrometer

The body of the hydrometer is partly immersed in the standard liquid with the stem still in air.

The calibration is made from the bottom of the scale to the top to avoid the build up of liquid drops on the stem. The stem is lowered into the liquid to the point at which it is to be calibrated.

5. Calibration – measurement

Care must be taken to go down sufficiently to have the graduation below the surface of the liquid, and to lift so as to build the meniscus systematically on a wet stem surface.

The reading is made with the graduation levelled to the main surface of the liquid.

The surface tension of the liquid must be well known for the correction to the calibration to be made for a corresponding zero value.

5. Calibration – volume

With the same set-up, it is possible to immerse the hydrometer completely and measure its density and volume.

Theory – balance calibration

Before starting a calibration, the balance must be exercised by loading it a few times with a weight.

Then it is calibrated, either by an internal weight or an external weight.

Theory
balance calibration

The calibration factor of the balance is

$$\beta = \left(1 - \frac{\rho_a^{'}}{\rho_{cal}} \right)$$

where

$\rho_a^{'}$ = air density during balance calibration

$\rho_{cal} = 8000 \ \text{kg/m}^3$ = density of calibration weight

Theory
first of three equations

Weighing of the hydrometer in air

$$\beta R_a = m - (V + v)\rho_a$$

where

R_a = balance reading of weighing in air

m = absolute mass of the hydrometer

$(V + v)$ = total volume of the hydrometer

$$\left(\begin{array}{l} V = \text{immersed volume of the hydrometer} \\ v = \text{non-immersed volume of the hydrometer} \end{array} \right)$$

ρ_a = density of air

Theory
second of three equations

Hydrometer floating freely in liquid at a scale reading

$$0 = m + \frac{\pi D \gamma_L}{g} - V \rho_L - v \rho_a$$

where

$D =$ diameter of the stem at scale reading

$\gamma_L =$ surface tension of liquid to be measured

$g =$ local acceleration of gravity

$\rho_L =$ density of liquid to be measured

Theory
third of three equations

Hydrometer suspended from balance in standard liquid

$$\beta R_s = m + \frac{\pi D \gamma_s}{g} - V \rho_s - v \rho_a$$

where

R_s = balance reading of suspended hydrometer

γ_s = surface tension of calibration liquid

ρ_s = density of calibration liquid

Theory
the three equations

$$\beta R_a = m - (V + v)\rho_a$$

$$0 = m + \frac{\pi D \gamma_L}{g} - V\rho_L - v\rho_a$$

$$\beta R_s = m + \frac{\pi D \gamma_s}{g} - V\rho_s - v\rho_a$$

Theory
solution of three equations
(simplified)

1) We neglect surface tensions γ

2) We set $m' = m - v\rho_a$ (to simplify the equations)

Theory
solution of three equations
(simplified)

$$\beta R_a = m' - V \rho_a$$

$$0 = m' - V \rho_L$$

$$\beta R_s = m' - V \rho_s$$

Theory
solution of three equations
(simplified)

$$\rho_L = \frac{R_a \rho_s - R_s \rho_a}{R_a - R_s}$$

(Neglecting surface tensions of liquids)

Theory
solution of three equations

But life is never that simple;

so, we take into account the surface tensions:

Theory
solution of three equations

$$\beta R_a = m - (V + v)\rho_a$$

$$0 = m + \frac{\pi D \gamma_L}{g} - V\rho_L - v\rho_a$$

$$\beta R_s = m + \frac{\pi D \gamma_s}{g} - V\rho_s - v\rho_a$$

Theory
solution of three equations

$$\rho_L = \frac{\beta\left(R_a \rho_s - R_s \rho_a\right) + \dfrac{\pi D}{g}\left(\gamma_s \rho_a + \gamma_L\left(\rho_s - \rho_a\right)\right)}{\beta\left(R_a - R_s\right) + \dfrac{\pi D}{g}\gamma_s}$$

Theory
solution of three equations

Remark: neglecting the surface tensions, we go back to previous simpler solution (in red)

$$\rho_L = \frac{\beta\left(R_a\rho_s - R_s\rho_a\right) + \dfrac{\pi D}{g}\left(\gamma_s\rho_a + \gamma_L\left(\rho_s - \rho_a\right)\right)}{\beta\left(R_a - R_s\right) + \dfrac{\pi D}{g}\gamma_s}$$

Theory
density of hydrometer

By immersing completely the hydrometer, it is possible to measure its density:

$$\rho_h = \frac{R_a \rho_s - R_s^c \rho_a}{R_a - R_s^c}$$

where

ρ_h = density of the hydrometer

R_s^c = balance reading at full immersion of the hydrometer

Theory
density of hydrometer

By immersing completely the hydrometer, it is possible to measure its absolute mass:

$$m = \frac{\beta R_a}{\left(1 - \dfrac{\rho_a}{\rho_h}\right)} = \frac{\beta R_s^c}{\left(1 - \dfrac{\rho_s}{\rho_h}\right)}$$

Theory
volume of hydrometer

By immersing completely the hydrometer, it is possible to measure its volume:

$$V + v = \frac{m}{\rho_h} = \frac{\beta R_a}{\rho_h - \rho_a} = \frac{\beta R_s^c}{\rho_h - \rho_s}$$

Conclusion

Advantages of the Cuckow method:

1. One liquid only. No dangerous and toxic liquids required for high density calibrations, like acids.

2. More accurate and faster than comparisons with reference liquids and reference hydrometers.

3. Calibrations can be made exactly at any scale marks on the stem.